Neutral Meson Production with Polarized X Rays*

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Measurements of π^0 photoproduction have been made at 235, 285, 335, and 435 MeV, using a beam of polarized x rays. Using a calculated value of polarization, an analysis is made which indicates a possible need for γ, ρ, π, or γ, ω, π coupling. The polarization calculations are checked by measurements made as a function of photon production angle at 335 MeV.

I. INTRODUCTION

THE polarized bremsstrahlung beam of the Stanford linear accelerator has been used to study π^0 photoproduction.

Measurements are made of the ratio R of the counting rates from meson production with the enhanced electric field vector perpendicular to the plane of meson production and in that plane. R is most conveniently related to the photon polarization P and the meson asymmetry A, by the relation

$$(R-1)/(R+1)=PA, \tag{1}$$

where

$$P=(N_t-N_r)/(N_t+N_r) \quad \text{and} \quad A=(\sigma_\perp-\sigma_{\parallel})/(\sigma_\perp+\sigma_{\parallel}).$$

Here N_t and N_r refer to the number of photons perpendicular and parallel to the plane of photon emission and $\sigma_\perp$ and $\sigma_{\parallel}$ refer to the differential cross sections for meson production with the electric field vector perpendicular and parallel to the plane of meson production.

Since a calculation of P is based on the principles of quantum electrodynamics and so should be accurate relative to R and A, we have used the measured values of R and calculated values of P to obtain the meson asymmetry A at the following points. See Table I.

These asymmetry values differ slightly from those published previously[1] since an error was found in the computer program used for calculating the polarization for the previously published data.

Because of the dominant 3,3 resonance near 335-MeV photon energy, a calculation of the asymmetry A should be more accurate at this energy than at others since the meson physics is best understood here. We have used the point taken at this energy as a check of the polarization calculation by comparing the calculated polarization with the polarization found from Eq. (1), using a calculated value of A. No significant disagreement was found. Additional data taken at 335-MeV photon energy showed that the polarization as a function of photon angle from the initial electron beam direction varied in a manner consistent with the polarization calculation.

Our asymmetry measurements have been combined with data from unpolarized experiments and examined for the effects of "retardation-like" terms due to the pion-pion resonances, analogous to the charged pion production retardation terms. The result is evidence for coupling between a photon, π meson, and pion-pion resonances, and indicates that the parameter Λ describing such coupling is positive in the sense that the dominating resonance must have a positive Λ.

TABLE I. π^0 photoproduction cross sections measured with polarized bremsstrahlung.

E_γ MeV	θ	$(\sigma_\perp-\sigma_{\parallel})/(\sigma_\perp+\sigma_{\parallel})$	P[a]
235	120°	0.289±0.047	0.15
285	90°	0.462±0.035	0.14
335	60°	0.462±0.025	0.16
435	90°	0.529±0.065	0.12

[a] These are averaged values of the photon polarization calculated for the experimental conditions at each energy.

II. PRODUCTION OF POLARIZED BREMSSTRAHLUNG BEAM

The same method of obtaining polarized bremsstrahlung has been used in this experiment as in the experiments of Taylor, Smith, and Mozley.[2,3] Bremsstrahlung produced at a small angle (mc^2/E) to the direction of the incident electron has a maximum of polarization tangential to a circle around the initial beam direction. By the use of a collimator and appropriate steering of the initial beam, the polarized region can be selected and varied. The polarization has been calculated by May[4] and also by Olsen and Maximon.[5] The formulas of May [Eqs. (2) and (3)] were used in these experiments but a check showed that they differed negligibly from the less approximate relation of Olsen and Maximon in the intervals used.

$$N_t=\frac{2\bar{\phi}}{\pi}\frac{d\epsilon}{\epsilon}\frac{d\varphi_0 dx_0}{(1+x_0)^2}\left\{\left[(1-\epsilon+\tfrac{1}{2}\epsilon^2)\ln\frac{1+x_0}{f}-(1-\epsilon)-\tfrac{1}{4}\epsilon^2\right]\right\}, \tag{2}$$

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[1] D. J. Drickey and R. F. Mozley, Phys. Rev. Letters 8, 291 (1962).

[2] R. E. Taylor and R. F. Mozley, Phys. Rev. **117**, 835 (1960).
[3] R. C. Smith and R. F. Mozley, Phys. Rev. **130**, 2429 (1963).
[4] M. M. May, Phys. Rev. 84, 265 (1951).
[5] H. Olsen and L. C. Maximon, Phys. Rev. **114**, 887 (1959).

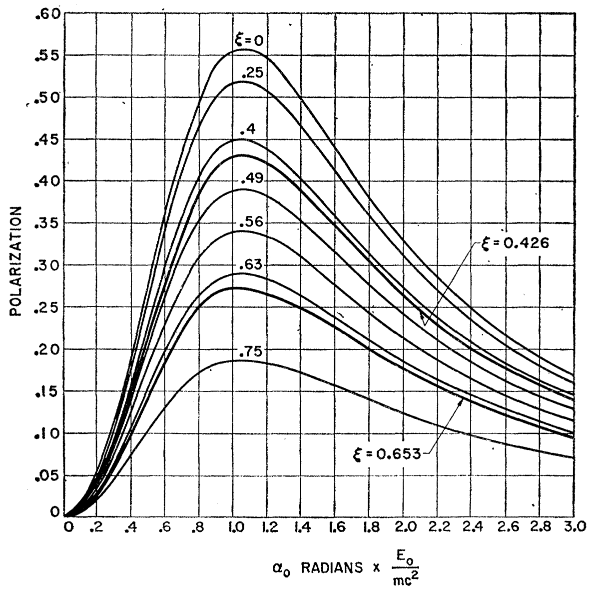

FIG. 1. Polarization as a function of angle for different values of photon energy, $\epsilon=k/E$.

$$N_r=\frac{2\bar{\phi}}{\pi}\frac{d\epsilon}{\epsilon}\frac{d\varphi_0 dx_0}{(1+x_0)^2}\left\{\left[1-\epsilon+\tfrac{1}{2}\epsilon^2-4(1-\epsilon)\frac{x_0}{(1+x_0)^2}\right]\times\ln\frac{1+x_0}{f}-\tfrac{1}{4}\epsilon^2+(1-\epsilon)\left[1-2\left(\frac{1-x_0}{1+x_0}\right)^2\right]\right\}. \quad (3)$$

$x_0=\epsilon^2\sin^2\theta_0$, $\epsilon=k/E$, $f=Z^{1/3}/108$, and $\bar{\varphi}=Z^2e^4/137$. E is the initial electron energy, φ_0 is the angle between the plane of the initial electron and photon and some fixed plane and θ_0 is the angle between the emitted photon and the initial electron.

These formulas do not take into account electron-electron bremsstrahlung, but a recent calculation of Scofield[6] shows this to be of essentially the same character, so its approximately 7% contribution in the case of the aluminum radiator used caused no appreciable error. The bremsstrahlung, as previously, was produced in the unanalyzed beam of the Stanford Linear Accelerator and hence, a very accurate control of the electron beam energy and energy width (approximately 3%) was not possible. The spectrum was monitored and a variation in energy or energy width of greater than 1% seems unlikely.

The radiation was produced from a 0.003-in. aluminum radiator (0.001 radiation length) and the region of polarization was selected by a $\frac{1}{4}$-in. collimator 40 ft distant. After passing through the radiator, the electron beam was deflected and its intensity and energy measured by secondary emission monitors. The intensity of the polarized beam was measured by a hydrogen ion chamber in the target area. Figure 1 shows the polarization expected as a function of angle before appropriate folds of multiple scattering, beam size, angular divergence and aperture were made. The angle could be varied by steering coils located immediately before the radiator. These coils could select regions of polarization in quadrature about the electron beam direction and hence vary the direction of polarization. This was done cyclically by switching the coils (and data storing scalers) after a predetermined amount of bremsstrahlung, the intervals being about a minute during approximately 40 h of data taking for each point, with data for each polarization stored in separate scalers. Beam-centering errors were reduced by the fact that at each polarization data were taken on either side of the central beam direction. The polarization increases for increasing electron energy and decreasing photon energy, but the maximum accelerator energy could not be used since the possibility of pion pair production had to be kinematically excluded.

As in the previous experiments, the polarization was determined by calculations. In this case the effects of multiple scattering and beam size and angular divergence were measured each data taking run, by measuring the size of an undeflected electron beam at the collimator after passing through a one-half thickness radiator replacing the usual one. This distribution was measured on a glass slide.

The only significant difference from previous experiments was that the $\frac{1}{4}$-in.-diam aluminum radiator used in the previous experiments to reduce the effects of poor beam focusing was, in much of this experiment, replaced by a continuous foil which intercepted the entire beam. The beam size was less than $\frac{1}{4}$ in., but in a portion of the experiment where the effects of polarization versus angle were studied, the steering coils were not sufficiently close to the radiator to prevent some movement of the beam off of the $\frac{1}{4}$-in.-diam central area. Although this movement did not change the polarization calculated, it would have done so if the entire beam had not struck the radiator.

Changes of the beam location and shape were checked visually by viewing the beam spot on a zinc sulfide screen with which the foil was replaced. During a single run, a maximum of 24 h, the spot location might drift a small amount, but the beam shape itself would not change unless an appreciable change was made in the accelerator adjustments. In all runs made at the same energy the polarization calculated from our measurements did not vary by more than $\pm 3\%$ or, for a typical value of polarization of 0.15, a change of 0.005. A comparison with the measured values of Smith and Mozley taken a year earlier indicated less than this variation. Hence we feel that our values were not too dependent on the accelerator conditions and we estimate an error in our calculated value of polarization of less than $\pm 2\%$ due to lack of control of the beam.

III. EXPERIMENTAL CONSIDERATIONS

The procedure used to obtain and monitor the polarized bremsstrahlung beam was described above.

[6] J. Scofield (private communication).

The equipment for production and detection of pions consisted essentially of a liquid-hydrogen target, a magnetic spectrometer, scintillation counter telescopes, and electronic equipment to analyze and record the phototube pulses.

The liquid-hydrogen target and the magnetic spectrometer were essentially those described by Smith where the hydrogen target was of the double-target condensation type with a large thick-walled reservoir serving as a coolant to condense and liquefy hydrogen gas from a ballast tank into one of the thin-walled targets. The second target was evacuated and used in the empty-target background runs. The target assembly could be moved vertically to place either the full target or the empty target in the beam as desired. The magnetic spectrometer was a 90°, 30-in. radius of curvature, 0.01-sr analyzing magnet placed on a shielding box made of 12-in.-thick steel plates. The spectrometer assembly could be rotated about the hydrogen target permitting measurements to be made at different angles. Thin Mylar windows on the magnet entrance and exit windows allowed the magnet aperture to be filled with helium gas to reduce proton energy losses. Slits consisting of 2 in. of copper followed by 6 in. of lead were mounted between the magnet opening and the target to restrict the polar angular opening of the magnet, while the azimuthal angles remained constant. A slit width of 4 in. was chosen as the largest consistent with the desired angular resolution.

Two counter telescopes located in the magnet focal plane were used to identify protons. Each telescope consisted of three counters, a $\frac{1}{8}$-in.-thick scintillator, a $\frac{1}{4}$-in.-thick scintillator mounted 4 in. below, and a 6-in.-thick Lucite Čerenkov counter directly below the second scintillator. Protons were identified by demanding large pulses in coincidence in the top two counters and no pulse in the bottom counter since the proton velocity was too low to produce Čerenkov light. Since the proton pulses were so much higher than pulses from other particles, nearly exact proton identification was possible with only one counter. This was especially true at the 335-MeV gamma-ray energy, 120° pion center-of-mass angle point. Here the proton momentum in the magnet (≈ 207 MeV/c) was too low for the particle to penetrate the top counter and still have enough energy to make a usable pulse in the bottom counter. For this reason this point was taken with three single counters instead of the counter telescopes. This method of particle identification was excellent at this point, since the cross section at resonance for π^0 production is much higher than backgrounds and competing processes. In addition, since this experiment involves only a ratio, it is not necessary either to maximize or even to know the efficiency of the counters, and discriminators could be set so high that they rejected some of the proton pulses.

Figure 2 is a block diagram of the electronic equipment used in the experiment considering only one counter telescope. The electronic circuits used for particle identification were modified copies of transistor circuits developed by Pine and Bazin with time resolution of about 15 nsec. Accidental rates in this experiment were found to be negligible.

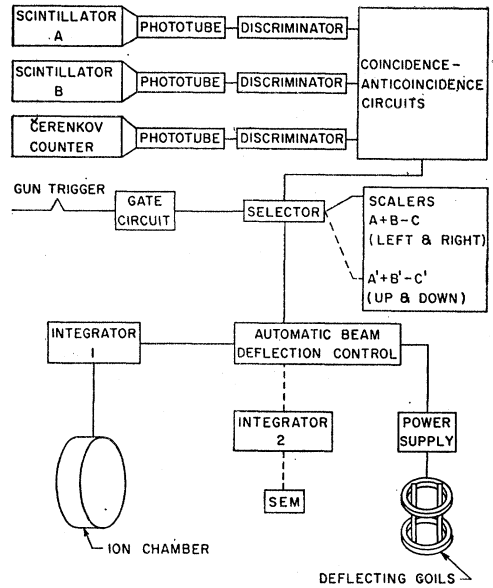

FIG. 2. Block diagram of the electronic equipment. One of three channels is shown.

Since π^0 photoproduction is a two-body reaction, determination of the energy and angle of the final proton specifies the kinematics completely. In practice the proton angle is specified to $\approx \pm 2°$. On the other hand, the central value of the proton energy and also the energy resolution are more difficult to determine.

Protons of these energies lose an appreciable fraction of their energy in the liquid hydrogen, target wall, heat shield, Mylar windows, and air and helium paths through the magnet. As a typical value, a 60-MeV proton loses ≈ 7 MeV in traversing this path. The energy centering procedure used consisted of calculating the momentum of the proton from the desired gamma-ray energy at a point halfway through the magnet and setting the magnet for this momentum. The major inaccuracies in this procedure are due to inadequate calculations of the protons' energy losses, improper magnet momentum calibration, variations in magnet shunt resistance since calibration, and incorrect location of the counters in the magnet focal plane.

Once this estimate of the proper magnet current setting was made, an excitation curve was run to determine the actual central gamma energy. Figure 3 shows such an excitation function for 285 MeV. The data consisted of the number of counts recorded for a constant amount of integrated ion chamber current at a given electron energy. Since the energy of the electrons striking our thin foil determined the peak bremsstrahlung energy, and since the shape of the brems-

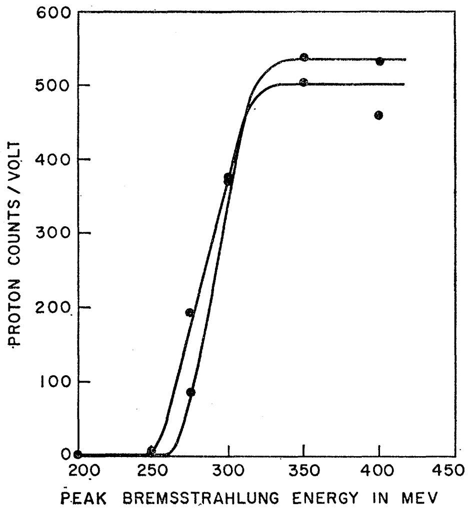

FIG. 3. Excitation function used to verify the correct spectrometer current to observe the π^0 photoproduction reaction at 285 MeV.

strahlung spectrum from a thin target is well understood, the central energy of gamma rays causing counts in our telescopes could be computed. In effect we averaged over energy-dependent and angle-dependent variables, such as the cross section, so that the counts above and below a given energy were equal.

Bremsstrahlung shape, electron energy spectrum, and ion chamber efficiency change as a function of energy must be folded into the resolution function of the apparatus to determine the true central energy from the excitation function.

Estimates show that the only important effect in this calculation is the bremsstrahlung shape. The accelerator energy spectrum, as displayed on the spectrum analyzer, is normally 2% full width at half-maximum. The energy variation at 300 MeV is then less than ± 3 MeV and was considered negligible in this fold. The ion chamber sensitivity change as a function of peak bremsstrahlung energy was also considered negligible over the energy variation used in the excitation functions. This conclusion is based on calibrations of similar ion chambers[7] and on quantitative estimates of the efficiency changes. The energy range in taking these excitation functions is not large and a thin-walled ion chamber should be sensitive primarily to the unchanged low-energy bremsstrahlung tail. As a result of these arguments the calculation is a fold of the bremsstrahlung shape into the magnet resolution function.

In performing the experiment we found the calculated magnet momentum and the momentum setting determined by the excitation function agreed to 5% or better. At the resonant energy point, 335-MeV gamma energy, 60° pion center-of-mass angle, the magnet current was corrected 5%. At all other points no correction was considered necessary.

[7] J. S. Pruitt and S. R. Domen, Natl. Bur. Std. (U. S.), Ann. Rept., 6218 (1958).

IV. BACKGROUNDS

A. General

Data were taken with full and empty targets and with and without the radiator foil, and appropriate subtractions made. At some energies the empty target background was negligible and not recorded.

Identification of the process $\gamma+p\rightarrow\pi^0+p$ by observing only the recoil proton is possible only as long as competing processes are small or kinematically avoided. Three competing processes were considered in this experiment: nuclear Compton effect, pion-pair production, and a double reaction discussed below. The nuclear Compton effect, $\gamma+p\rightarrow\gamma+p$, has kinematics so similar to pion photoproduction that in practice the kinematics could not be used to eliminate it. Fortunately its cross section is small compared to photoproduction, probably less than 1%, and can be considered negligible. Pion-pair production reactions such as $\gamma+p\rightarrow 2\pi+p$, have a cross section nearly equal to single pion production and so cannot be neglected but fortunately can be eliminated by kinematics at all but the 435-MeV point. Here we decided to correct the data to account for the competing process.

B. Pion-Pair Production

Since the polarization at a given bremsstrahlung energy increases as the electron beam energy increases, the electron energy must be as high as possible. The energy cannot be too high or protons from pion-pair production contaminate the data. The optimum point to take data thus corresponds to the condition that protons from pion-pair production from gamma rays of the maximum energy of the bremsstrahlung are barely excluded from the counting system.

The pion center-of-mass angles for each point in this experiment were chosen by considering the particular experimental conditions at each point. Since asymmetry effects in π^0 photoproduction are largest at 90° and decrease to zero at 0° and at 180°, the angles were chosen as near 90° as possible. At 235 MeV, the angle 120° was chosen since it was the smallest angle at which the recoil proton had sufficient momentum to escape the target with only a small energy loss and be analyzed and counted in the spectrometer system. At 285 MeV, it was possible to run at 90°. At 335 MeV, relative polarization points were taken at 90° but pion-pair contamination could be avoided only by going to 60°.

π-pair contamination was excluded for all but the 435-MeV point. Here the machine energy so determined would have been sufficiently near the gamma-ray energy being observed in π^0 production that the polarization would have been too low to be useful. For this reason a beam energy of 575 MeV was more or less arbitrarily chosen and the data corrected for pion pairs. The published data on pion pair production were insufficient to allow us to estimate a correction for our data and hence we were compelled to measure the amount of contamination. This measurement was made with different experimental equipment since the energy and angular resolution of our own was inadequate. In particular we used a high-resolution 1-BeV/c spectrometer[8] and different electronic equipment.[9] In addition, with this experimental arrangement the electron beam could be energy analyzed before it struck the radiator permitting a more accurate ($\frac{1}{2}\%$) definition of the peak energy of the bremsstrahlung. Finally this equipment was preferable to ours since our beam monitoring devices are adequate for taking ratios but are unreliable for the absolute measurements necessary to evaluate the pion-pair correction.

The method used was to measure the yield of protons from a photon beam striking a liquid-hydrogen target as a function of peak bremsstrahlung energy. The spectrometer was set so as to observe protons of momentum and angle corresponding to π^0 photoproduction at 435 MeV, 90° center-of-mass angle. As the peak bremsstrahlung energy is raised above pion-pair production threshold, one should observe an increase in the counting rate above counts due to single pion photoproduction. Counts were taken in 25-MeV steps from 450 to 625 MeV (the upper limit being 50 MeV above our peak bremsstrahlung energy). Three runs at each energy were taken corresponding to target full, radiator in; target full, radiator out; and target empty, radiator in conditions. A radiator out, target out run produced so few counts that it was neglected. The beam intensity was monitored by using a secondary emission monitor (S. E. M.) to measure the electron beam intensity before it struck the radiator. The beam was "swept" by a deflecting magnet after it struck the radiator in order to remove charged particle contamination. The S. E. M. was calibrated at various energies by comparing it with a Faraday cup used to collect the electron beam. Since the Faraday cup monitor was independent of energy, any variation in the ratio of the two monitors must be due to the S. E. M. The maximum observed variation of the ratio, 0.9%, was small enough to be neglected in analyzing the data. Counting rate corrections were also small and hence neglected because they produced only a minor effect on the radiator in, target in, run. Gamma beam position was centered on the target by observing the beam position on a cesium bromide crystal placed in the beam line directly after the hydrogen target.

The data were analyzed by normalizing and subtracting the two background runs and the data were then normalized to a constant integrated current from the S. E. M. at each energy. Since the bremsstrahlung shape influences the counting rate, the above counting rates at each energy were folded into the number of

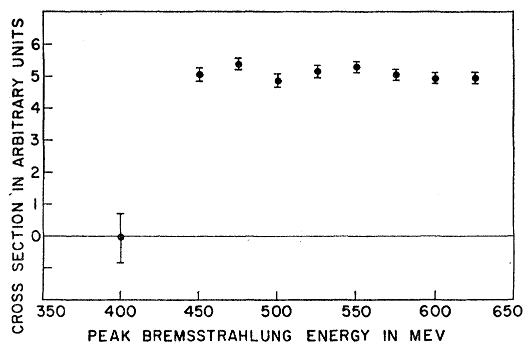

FIG. 4. Excitation function to check the size of the pion-pair background which was not kinematically excluded.

photons per energy interval at 435 MeV. The values of the number of photons per energy interval were computed taking into account the radiator thickness. The radiator used in the measurement was $\frac{1}{16}$-in. aluminum so that only minor deviations from the thin radiator spectrum were found. The result of this fold is a number proportional to the cross section assuming all counts come from the π^0 reaction only. If the pion-pair reaction contributes, the points above pion-pair threshold at 560 MeV should be larger than the points below this threshold.

Figure 4 shows the resulting data. The conclusion is that the numbers are equal within the $\approx 3\%$ statistics so that the assumption of no pion-pair contamination is valid for these experimental conditions. Since the conclusion is only valid within the statistics, $(R-1)/(R+1)$ at the 435-MeV point has assigned to it an additional 3% error to take account of the unknown pion-pair contamination.

It is interesting to point out that our above conclusion about the pion-pair-production cross section is consistent with data taken at other angles and energies by Richter.[10]

C. Double Reaction Background

Our experiment was intended as a study of π^0 photoproduction near the resonance region, but during the course of the experiment we decided to attempt a preliminary investigation of the above resonance region, i.e., the region above 450-MeV gamma-ray energy, to determine the experimental problems there and to see if the present equipment would be useful for such measurements.

Among the measurements made was an excitation function taken near 700-MeV gamma-ray energy observing 360 MeV/c recoil protons at 62° laboratory angle. We found a surprisingly high background even at energies well below single-pion threshold. (See Fig. 5.) As this figure shows, the counting rate did not drop to zero until the peak beam energy was dropped to 300–350 MeV, the 33 resonance region.

[8] R. Hofstadter, F. A. Bumiller, B. R. Chambers, and M. G. Croissiaux, *Proceedings of an International Conference on Instrumentation for High-Energy Physics, Lawrence Radiation Laboratory, 1960* (Interscience Publishers, Inc., New York, 1960), p. 311.

[9] C. Schaerf (to be published).

[10] B. Richter (private communication).

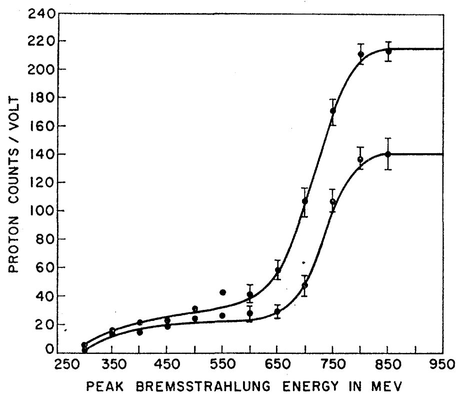

FIG. 5. Excitation function at 725 MeV. The double reaction background is shown.

We investigated this surprising background rather extensively in spite of the low counting rates involved in order to attempt to understand its sources and in order to be sure that it would not influence our lower energy data. These investigations revealed the following properties of the background.

(1) The particles are protons of ≈ 360 MeV/c. The proof for this statement rests both on the elimination of π^+ mesons and the identification of protons. 360-MeV/c π^+ mesons were eliminated by noting that when the beam energy was dropped below 475 MeV, the π^+ meson threshold for these kinematic conditions, the Čerenkov counting rate vanished, but the unknown background in the scintillators persisted. The particles were identified as 360-MeV/c protons because they gave large proton-like pulses and because $\frac{1}{4}$-in.-CH absorber did not stop them, but $\frac{3}{8}$-in. Al did. The range of an 82-MeV/c π^+ is 4.3-g/cm^2 Al, but such a pion at this angle must come from a gamma energy of less than 200 MeV while the background is zero at 300–350 MeV. The addition of baffles to the magnet did not appreciably change the background rate relative to the π^0 rate giving additional confidence that lower energy particles were not scattering through the magnet. The particles must come through the magnet aperture because blocking the aperture eliminated the background. Heavier particles, for example deuterons, were not eliminated but the pulse heights determined with a 256-channel analyzer showed a negligible number of pulses higher than the proton peak.

(2) The background is not protons from photodisintegration of some heavy nucleus. This statement follows from the fact that all sources of photoprotons were investigated and found to be small. In particular, the hydrogen gas was analyzed using a mass spectrograph and found to contain less than 0.03% D_2 and less than 0.01% heavier contaminants. Such small concentrations cannot produce the observed counting rates. Target wall photoprotons were trivially eliminated by an empty target run; in this case any possible variation in the dual targets was eliminated by using the target normally filled with hydrogen for this run. The background is not from the proton Compton effect, because these are kinematically eliminated at ≈ 577 MeV.

(3) The background is not from charged contaminants in the gamma-ray beam. A possible candidate for the source of the background is the reaction $e+p \rightarrow e+p$ where the initial electrons are contaminants in the beam. When the beam was swept by a magnetic field sufficient to deflect even 700-MeV electrons so they would miss the target and scattering chamber, the background remained unchanged. Also the insertion of a 0.5-in. aluminum radiator into the gamma-ray beam did not increase the background as this hypothesis would indicate. All these statements it must be remembered are true within the rather poor statistics taken in these checks.

(4) The background is the same at 40 and 60° laboratory angles and decreases with increasing proton momentum. The final series of runs taken to investigate this background revealed the background at 360 MeV/c was constant within statistics between 90 and 40° lab angle. With a $\frac{7}{8}$-in. Al energy degrader in front of the magnet entrance in a relatively "bad geometry" to permit measurement of 450-MeV/c protons, the background was reduced to 40% of the 360 MeV/c rate at 90 and at 56°. At 275 MeV/c, 90° lab angle, the background was 30% higher than at 360 MeV/c and decreased with increasing momentum as shown in Table II.

(5) The background was negligible at our low-energy points and at 435-MeV photon energy and 90° c.m. angle. The point in our experiment most sensitive to this background was the one at 235-MeV photon energy. At the other points the π^0 cross section was so large it would dominate any background effect of this nature. At the 235-MeV point the peak bremsstrahlung was only 335 MeV. Measurement of the counting rate at 225 MeV/c and 25° lab angle (corresponding to photoproduction at 235 MeV), and a comparison with the background found at a larger angle indicated that the background was 1% with the beam at 350 MeV and 0.3% with the beam at 300 MeV, assuming it is independent of angle as previously shown and hence it is negligible. In performing the excitation curve at the 435-MeV point with 7/8-in. Al over the magnet entrance in order to observe 450-MeV/c protons (this corresponds to 90° c. m. angle for π^0 photoproduction) the background was found to be $\approx 2\%$ and was also considered negligible.

TABLE II. Variation of double reaction background with energy.

Proton "energy"	Counts/Unit beam intensity
225 MeV/c	37
225 MeV/c+$\frac{1}{4}$-in. Al	22
225 MeV/c+$\frac{5}{8}$-in. Al	14
225 MeV/c+$\frac{7}{8}$-in. Al	6

Examination of several hypotheses left the following candidate as a source for the background. It is probably due to a double reaction: $\gamma+p\rightarrow\pi^{+}+n$ followed by $\pi^{+}+p\rightarrow\pi^{+}+p$. The initial reaction occurs in the Mylar target walls and liquid hydrogen directly in the beam line. The second takes place in the liquid hydrogen or in any of the surrounding material that such a π^{+} meson might strike and still scatter a proton into the magnet. The protons, of course, are free only if the reaction takes place in the hydrogen. Other double reactions can take place, but estimates indicate this is the major one and at least serves to illustrate the type of reaction involved.

The reason this particular hypothesis is so attractive is the fact that the first process, π^{+} photoproduction, has a large cross section in the region of 300–400 MeV, the energy region that appears to be the cause of the background. Of course, π^{0} photoproduction also has a large resonance in this region but by examining the kinematics one finds that only a light particle (π^{+}) can scatter a heavy particle (proton) at a large angle so as to cause this background. Of course the recoil proton from π^{0} production can scatter from heavier elements such as those in the target walls and enter the magnet, just as the recoil nuetron from π^{+} photoproduction can charge exchange scatter from these nuclei. Secondary reactions from the π^{0} decay rays can also contribute but are surely small since the second reaction is electromagnetic.

The preceding considerations indicate that the background has been sufficiently well investigated to prove it does not influence the results of this experiment.

V. ERRORS

The errors appended to our final data are obtained entirely from counting statistics. There are, of course, many other sources of systematic error than those discussed above in the determination of the value of the polarization.

Expression of the data as a ratio, however, has the useful property of reducing the effects of systematic errors such as improper evaluation of backgrounds or counting rate corrections. For example, let $R=(A+a)/(B+b)$ where A and B are the true values and a and b small systematic errors. Then $R\simeq(A/B)(1+a/A-b/B)$ and to the extent that the proportional errors are the same they have no effect on the ratio.

In addition, since identical counting equipment was used for meson detection with both photon polarizations the counter efficiency will not appear in the ratio if one can rely on the stability of the equipment. Since the polarization was cycled at intervals of about 1 min we feel that such changes can be neglected in relation to our counting statistics.

The energies and angles are the central values of these variables. The equipment used in this experiment defined photon energies to about $\pm10\%$ and laboratory angles to about $\pm2°$. The angle θ of Table I is, of course, the center-of-mass angle of the π^{0} meson. The central values of photon energy E_{γ} are uncertain to ≈4.5 MeV estimated by the amount of disagreement between the energy determined by calculating the proton recoil momentum, the energy determined by the excitation function, and the errors in the excitation function analysis. Laboratory angles were determined to $\approx0.2°$. The spectrometer current is reproducible to 0.1% and the angle setting is reproducible to 0.05°. If the target position is unchanged so that proton momentum losses are unchanged, the central photon energy should be constant from night to night. Target positioning was checked several times during the course of the experiment by taking an x-ray photograph of the beam position and determining this position relative to the target. No significant change was noted. Such a check was necessary because the target was removed from the beam line several times during the experiment.

VI. THEORETICAL CONSIDERATIONS

A. General

It has generally been considered that unless some type of peripheral production process occurs, meson photoproduction at and below the region of the first resonance should be well described by an analysis containing only S and P waves. Dispersion theoretical analyses show that in the case of π^{0} photoproduction the D wave is due to a term from the recoil of the proton, and hence, one might expect it to be small at low energies.

Under these assumptions the differential cross section can be expressed phenomenologically as follows[11]:

$$\begin{aligned}\sigma=K\,\mathrm{Re}\big[&|E_{01}|^2+|M_{11}|^2+\tfrac{5}{2}|M_{13}|^2+\tfrac{9}{2}|E_{13}|^2\\&+(M_{11}{}^*M_{13})+3(M_{11}{}^*E_{13})\big]-3(M_{13}{}^*E_{13})\\&+\cos\theta\big[-4(E_{01}{}^*M_{11})+2(E_{01}{}^*M_{13})+6(E_{01}{}^*E_{13})\big]\\&+\cos^2\theta\big[-\tfrac{3}{2}|M_{13}|^2+\tfrac{9}{2}|E_{13}|^2-3(M_{11}{}^*M_{13})\\&-9(M_{11}{}^*E_{13})+9(M_{13}{}^*E_{13})\big]\\&+\sin^2\theta\cos2\varphi\big[-\tfrac{3}{2}|M_{13}|^2+\tfrac{9}{2}|E_{13}|^2-3(M_{11}{}^*M_{13})\\&+3(M_{11}{}^*E_{13})-3(M_{13}{}^*E_{13})\big],\end{aligned}\tag{4}$$

or

$$\sigma=A+B\cos\theta+C\cos^2\theta+\alpha\sin^2\theta\cos2\varphi.\tag{5}$$

In this expression K represents phase space and normalization factors, θ is the pion center-of-mass angle, φ is the angle between the plane of photon polarization and the plane of pion emission, and E_{ij} and M_{ij} are the electric and magnetic multipoles where i is the relative angular momentum, and j is twice the total angular momentum of the pion and proton. The presence of D-wave terms will cause the addition of angular terms in $\cos^3\theta$ and $\cos^4\theta$ while terms in $\cos\theta$ and $\cos^2\theta$ will be added to the polarization asymmetry term. In the region of interest in this experiment between 60 and 120°, the neglect of these terms does not

[11] M. J. Moravcsik, lectures given at Purdue University, 1957 (unpublished).

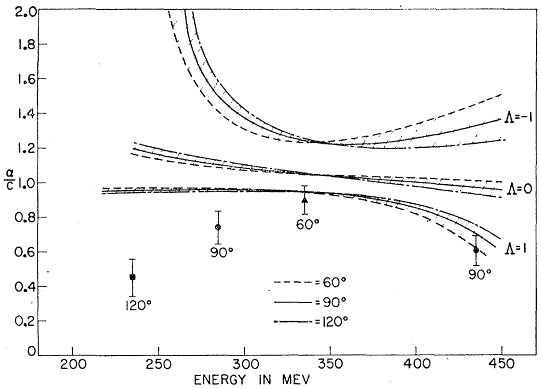

FIG. 6. α/C as a function of energy and Λ from McKinley theory (see Ref. 15). The bands show the variation between 60 and 120° pion angles. Experimental values are given.

cause very large errors, but in our calculations of the A, B, and C coefficients we have analyzed theoretical cross sections to obtain a best fit using only two powers of $\cos\theta$. This is done in a somewhat experimental manner giving equal weight to all angular "measurements" and using angular intervals of 15°.

It can be shown[12] that if, as seems approximately true, the data analyzed are symmetrical in the size of their errors at angles on either side of 90°, the presence of a $\cos^3\theta$ term which is ignored in the analysis will change only the size of the B coefficient and not affect the A or C terms. Similarly, a $\cos^4\theta$ term would affect only the coefficients of even powered terms. The change of the coefficient is dependent on the weighting of the experimental data.

Phenomenologically much is known about the π^0 photoproduction matrix element. The resonance at 325–350-MeV photon energy is magnetic dipole P wave with M_{13} dominating. Angular distribution studies show the D wave is small at these energies as expected. The coefficient B is found experimentally to be small between threshold and resonance which implies that electric dipole production is small.

If D wave is negligible, then from Eqs. (4) and (5), it can be seen that if in addition E_{13} is small, $\alpha=C$. This results from a purely phenomenological analysis using only the above two assumptions. The consequences of this equality produce interesting results concerning the meson asymmetries. Equation (5) in terms of polarized cross sections ($\sigma_\perp$ corresponds to $\varphi=\frac{1}{2}\pi$, $\sigma_{\parallel}$ to $\varphi=0$) may be written

$$(\sigma_\perp-\sigma_{\parallel})/(\sigma_\perp+\sigma_{\parallel})=-\alpha\sin^2\theta/\sigma_0\,, \tag{6}$$

where σ_0 is the unpolarized differential cross section. But this implies:

$$\frac{1}{P}\left(\frac{R-1}{R+1}\right)=-\frac{\alpha\sin^2\theta}{\sigma_0} \tag{7}$$

[12] Pierre Noyes (private communication).

which may be written

$$\frac{\alpha}{C}=-\frac{1}{P}\left(\frac{R-1}{R+1}\right)\frac{1}{\sin^2\theta}\left(\frac{A}{C}+\frac{B}{C}\cos\theta+\cos^2\theta\right). \tag{8}$$

Thus we have an equation for α/C which, if the preceding assumptions are correct, must equal one.[13] The equation involves observed ratios $(R-1)/(R+1)$, calculated values of polarization P, and angular coefficients A, B, and C determined in conventional experiments with unpolarized γ rays. Unfortunately although calculations using the Chew, Goldberger, Low, and Nambu dispersion theory (CGLN)[14] are in good accord with such an assumption, the less approximate estimate of McKinley[15] seems to be in much worse agreement. His approach is similar to that of CGLN except that all quantities are kept in a relativistic form rather than expanded in powers of $1/M$, and the method by which unitarity is satisfied is more general.

Using the McKinley theory we obtain the values of α/C shown in Fig. 6 ($\Lambda=0$). One can infer only that the E_{13} and D-wave terms are by no means dominant and that their contributions are such as to increase α/C below resonance. As a result we can use phenomenological arguments only to point out that α/C should not differ greatly from 1 if no ρ or ω coupling are included.

A recent calculation by Gourdin and Salin,[16] which will be considered in more detail below, produces a large electric quadrupole contribution, but we feel that this calculation is inconsistent with our measurements for other reasons.

In dispersion theoretical analyses of π^0 photoproduction it is found that at low energies the cross sections obtained are very dependent on the values of the phase shifts used. The errors in the measurements of the phase shifts are large enough to allow a reasonable fit to most photoproduction data if the phase shifts are treated as free parameters within slightly more than reasonable limits. A major difficulty is that it is impossible to fit both π^+ and π^0 photoproduction with the same set of phase shifts.

No recent complete analysis has been made of all of the scattering data to obtain a set of phase shifts valid over the entire energy region below 400 MeV. McKinley[15] has analyzed the scattering data in an incomplete manner by accumulating all of the results of scattering phase-shift analyses and obtaining interpolation formulas which give best fits to these data. As he points out, the scattering analyses will have coupled errors which will not be taken into account

[13] We are indebted to L. Koester for suggesting this type of analysis.

[14] G. F. Chew, M. L. Goldberger, E. F. Low, and Y. Nambu, Phys. Rev. **106**, 1345 (1957).

[15] J. M. McKinley, Technical Report No. 38, Physics Department, University of Illinois (unpublished); and Rev. Mod. Phys. **35**, 788 (1963).

[16] M. Gourdin and P. Salin, Nuovo Cimento **27**, 193 (1963); P. Salin, *ibid.* **28**, 1294 (1963).

TABLE III. Phase shift values used in these calculations, taken from McKinley.[a]

Photon energy (MeV)	δ_1	δ_3	Phase shift (degrees) δ_{11}	δ_{13}	δ_{31}	δ_{33}
235	8.60	−8.35	−0.72	−0.26	−2.38	17.8
285	10.3	−12.2	−0.96	−0.55	−3.43	43.7
335	12.5	−16.1	−0.58	−0.94	−4.15	82.1
435	21.3	−23.0	3.65	−1.99	−5.19	127.9

[a] See Ref. 15.

by this analysis. However, this is the only phase-shift set which encompasses our energy region and we use his values in the analysis of our data. See Table III.

Table IV shows the results of these calculations at 235 MeV with various values of the phase shifts. It can be seen that α/C and C/σ are sensitive to the phase shift values but that α/C is much less affected. Moreover, for any reasonable value of the phase shifts α/C is greater than 1. On the other hand, if we introduce the pion-pion resonances, a large variation of α/C is possible.

TABLE IV. Variation of α/σ_0, C/σ_0, and α/C with phase-shift values at 235 MeV. The values of Table III are used except for the phase shift tabulated in the left column. Changes in δ_1 and δ_3 cause negligible effects.

Phase shift in degrees	$-\alpha/\sigma_0$	$-C/\sigma_0$	α/C
Normal (See Table III)	0.3768	0.3145	1.20
$\delta_{11}=1.28$	0.4426	0.3730	1.19
$\delta_{11}=-2.72$	0.3074	0.2528	1.21
$\delta_{13}=1.74$	0.3834	0.3156	1.22
$\delta_{13}=-2.26$	0.3698	0.3169	1.17
$\delta_{31}=-0.380$	0.4876	0.4499	1.09
$\delta_{31}=-4.38$	0.2608	0.1684	1.55
$\delta_{33}=19.8$	0.4030	0.3482	1.16
$\delta_{33}=15.8$	0.3418	0.2686	1.27

B. Dispersion Theory Analysis Including ρ and ω Resonances

The fact that α/C might be less than one at points below resonance has been pointed out by De Tollis and Verganelakis.[17] They consider the contribution to neutral pion photoproduction of the two diagrams of Fig. 7 which in essence add to the CGLN[14] dispersion theory contributions of the form of retardation terms which they obtain by treating the resonances as particles. The resulting amplitudes have free parameters Λ_ρ and Λ_ω which take into account the strength of the γ, ρ, π, and γ, ω, π vertices.

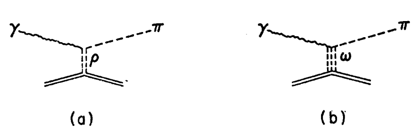

FIG. 7. Diagrams used in the analysis of De Tollis and Verganelakis to predict $\alpha/C<1$ below resonance.

[17] B. De Tollis and A. Verganelakis, Nuovo Cimento **22**, 406 (1961).

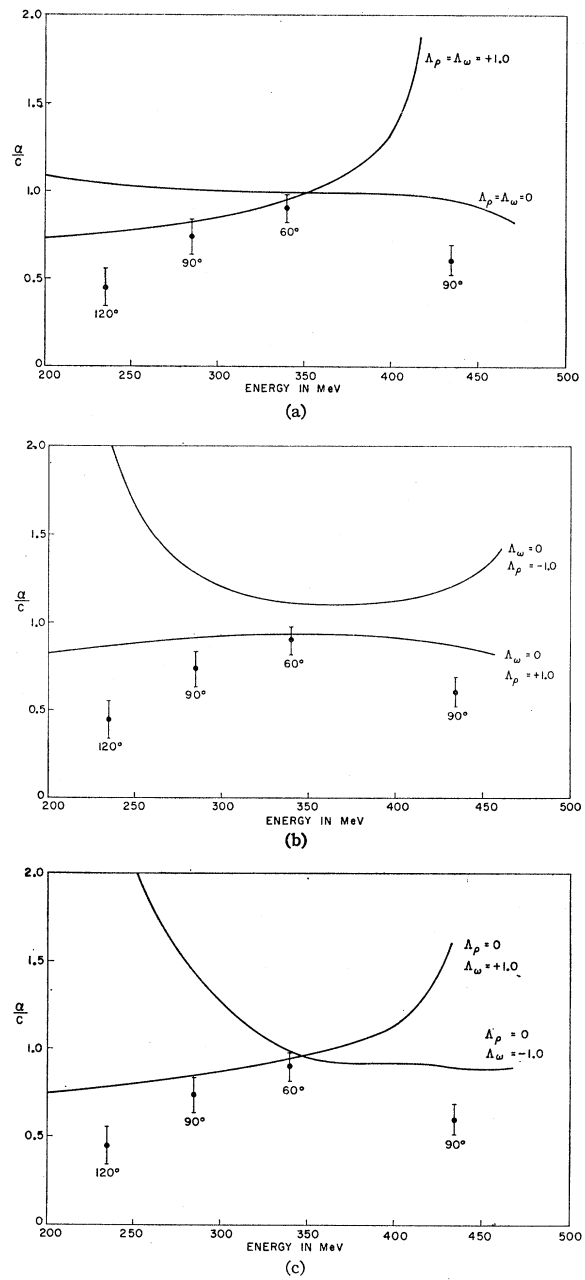

FIG. 8. α/C versus energy for various values of the bipion coupling constants calculated from CGLN using the McKinley phase shifts (see Ref. 15). Experimental values are shown. (a) $\Lambda_\rho=\Lambda_\omega=+1.0$ and $\Lambda_\rho=\Lambda_\omega=0$; (b) $\Lambda_\omega=0$, $\Lambda_\rho=+1.0$ and $\Lambda_\omega=0$, $\Lambda_\rho=-1.0$; (c) $\Lambda_\rho=0$, $\Lambda_\omega=+1.0$ and $\Lambda_\rho=0$, $\Lambda_\omega=-1.0$.

McKinley has included the γ, ρ, π coupling in his calculations. In the De Tollis and Verganelakis analysis the ρ and ω coupling produced little effect at resonance. Unfortunately the energy variation of the ω term is qualitatively different from that of the ρ term as can be seen from Figs. 8(a), (b), and (c) where we show the energy dependence of the ratio α/C as calculated from CGLN theory, for various values of the pion-pion coupling constants. We have used the phase shifts of McKinley for these calculations to permit a better comparison of the two theories, the calculations using McKinley theory being shown in Fig. 6. Since the more accurate McKinley theory does not include the ω, we have used CGLN to indicate the effect of these terms. In the De Tollis and Verganelakis analysis the ρ and ω coupling produced little effect at resonance because of the "crossover" effect caused by the large ω contribution above resonance. Presumably the same effect would be produced by including the ω term in the McKinley theory. McKinley points out that the theory including the ω term may be inaccurate because the ω, which contributes in F^+, can be rescattered through the 3,3 resonant state.

Terms of this retardation-like nature obviously contain higher angular momenta and an analysis of data in terms of only two powers of $\cos\theta$ must be inexact. Since the experimental data which we use were analyzed in terms of only two powers, it is not useful for us to analyze our data in terms of a theoretical C coefficient, and the values we use are of a best fit nature as described above. In cases where the Λ parameter is large, this best fit is in fact not very good. The α coefficient also has an angular dependence and in this case we use the actual value in comparisons with our data.

It is interesting that α/σ_0 is less sensitive to the pion-pion terms than C/σ_0. This is true because the dominant terms in Eq. (4) are E_{01} and M_{11}. But C and α have the form

$$C=k\mathrm{Re}[Y+Z+3E_{13}^*(M_{11}-M_{13})], \qquad (9)$$
$$\alpha=k\mathrm{Re}[Y-Z-E_{13}^*(M_{11}-M_{13})],$$

where Y and Z represent terms common to both α and C; since the pion-pion terms produce large changes in E_{13} and M_{11}, their effect is much larger on C than on α. The term $E_{13}^*M_{13}$ is large, of course, because M_{13} is the main resonant term in photoproduction.

VII. EXPERIMENTAL RESULTS

Two separate types of measurements were made. In one the meson asymmetry was studied as a function of energy using calculated values of the polarization. If one suspects the validity of such a calculation, one would expect, however, that the characteristics of the change of asymmetry with energy should be valid even if the absolute value of the polarization were wrong.

In the second type of measurement we use the known meson asymmetry to study the polarization. Such a double experiment is not completely fatuous since we use the well understood meson production at resonance for this determination. It had been our belief that phenomenological arguments such as those advanced earlier made such an attempt reasonable. Earlier calculations using CGLN had shown that the electric quadrupole and D-wave terms were small and thus we felt relatively independent of minor inaccuracies of the dispersion theory. Moreover, the possible presence of the ρ or ω coupling seemed unimportant at resonance since these terms are 90° out of phase with the main $\frac{3}{2}$, $\frac{3}{2}$ term at this point. However, as pointed out above, a calculation using the McKinley theory shows that the assumptions are not completely valid, and in particular, introduction of the γ, ρ, π coupling produces a contribution which, although a minimum at resonance, is not negligible. As a result, we can use the phenomenological arguments and the variation of α/C with pion-pion term merely to give a feel for the possible error made by assuming that $\alpha/C=1$ at resonance. An accuracy of about 10% might be justified for this assumption.

In addition, a study was made of the change of polarization with bremsstrahlung angle, and in this case our results are independent of our knowledge of the exact value of the polarization.

A. Analysis of the Meson Asymmetry Measurements

Table I shows the results of our measurements of meson asymmetry as a function of photon energy using the computed values of polarization. The errors quoted are statistical only, and are a combination of the errors in $(R-1)/(R+1)$, and the errors in P determined from the errors in making the glass slide measurements of the multiple scattering electron distribution. Other errors that might influence the results have already been discussed and the arguments indicate that they should be small.

The previous analysis predicts that α/C should be approximately one near resonance if no pion-pion coupling term exists. Consequently it is of interest to combine our measured asymmetries with the results of angular distribution measurements to determine the ratio α/C.

The data analyzed in this manner using Eq. (8) are shown in Figs. 6 and 8 and show a striking disagreement with $|\alpha/C|=1$ at the low-energy points and reasonable agreement at the resonance point. The values used for the angular coefficients were taken from Berkelman and Waggoner[18] at the high-energy points and from Vasilkov *et al.*[19] at the low-energy 235-MeV point. Angular coefficients were interpolated to the proper energy by a smooth fit to the data in each paper, and the error assigned to each coefficient was the error in the coefficient at the nearest point. In evaluating Eq. (8), B/C was assigned zero error since B is small, and the error in A and the much larger error in C were

[18] K. Berkelman and J. Waggoner, Phys. Rev. **117**, 1364 (1960).
[19] R. Vasilkov, B. Govorkov and V. Goldanski, Zh. Eksperim. i Teor. Fiz. **37**, 11 (1959) [English transl.: Soviet Phys.—JETP **10**, 7 (1960)].

TABLE V. Experimental angular coefficients (Refs. 17 and 18) and resultant α/C ratio as a function of energy.

Energy (MeV) γ	A	B	C	α/C
235	7.4 ± 0.2	-0.8	-5.2 ± 0.6	0.452 ± 0.108
285	20.7 ± 0.6	-0.6 ± 0.5	-14.2 ± 1.5	0.740 ± 0.097
335	26.1 ± 0.4	0.7 ± 0.7	-16.3 ± 1.0	0.900 ± 0.081
435	9.2 ± 0.2	1.0 ± 0.3	-8.0 ± 0.5	0.608 ± 0.085

combined statistically. It was felt that this estimate was sufficiently conservative for our purposes and that the uncertainties in measurements of the angular coefficients were so large that a more sophisticated analysis was not justified. (See, for example, Berkelman and Waggoner.)

Table V lists the angular coefficients used in computing the results of Fig. 7 with the resultant values of α/C. It should be pointed out that the largest error in computing α/C comes from the errors in A and C.

It can be seen from Fig. 8 that even if our calculated value of polarization is incorrect, it is doubtful that our conclusions are changed since α/C is less than one at points below resonance and approximately one at resonance. This can be seen from the following argument: If the polarization is normalized, assuming the phenomenological assumptions to be correct, so that $\alpha/C=1$ at resonance and the polarization at the other points correspondingly changed, the resultant shift is too small to make the point at 235 MeV consistent with one. (Such a normalization would be an approximately constant shift of the points of Fig. 8 so that the 335-MeV point lies on the line $\alpha/C=1$.) In this sense our data do not depend on an absolute knowledge of the polarization. Although other normalizations of α/C are possible, they should not differ much from theoretical predictions, and hence, the variation of our measured asymmetry with energy appears inconsistent with a theory which does not include ρ or ω coupling.

It is possible that the published value of $-C/\sigma_0$ for low energies may be too large or its error too small. Experiments of Bulos[20] at Stanford at energies lower than 235 MeV seem to give some evidence that $-C/\sigma_0$ may be smaller than quoted by Vasilkov *et al.* These new data are tentative at present and have rather larger errors attached to the C coefficient so that the disagreement may not be conclusive. In addition, unpublished results of Modesitt[21] give smaller values of C at 235 MeV, and would lead to $\alpha/C\simeq0.6$ at this point.

We shall use published data keeping in mind the need of an additional measurement at this energy region.

Dispersion theory analysis of Sec. VI indicates that α/C can be made less than one below resonance if pion-pion terms are present with positive Λ. A difficulty

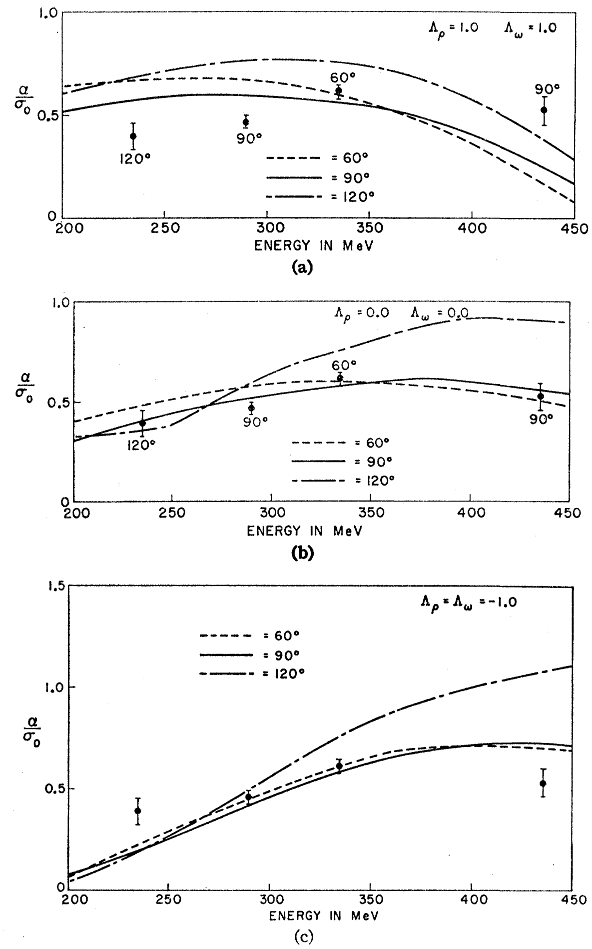

FIG. 9. $-\alpha/\sigma_0[=(\sigma_\perp-\sigma_{\parallel}/\sigma_\perp+\sigma_{\parallel})(1/\sin^2\theta)]$ versus energy for $\theta=60$, 90, and 120°. (a) $\Lambda_\rho=\Lambda_\omega=1.0$, (b) $\Lambda_\rho=\Lambda_\omega=0$, (c) $\Lambda_\omega=\Lambda_\rho=-1.0$. In this plot the effects of considering the ρ or the ω separately are qualitatively similar as a function of energy.

in an accurate analysis is that the results below resonance are sensitive to the small P-wave phase shifts which are not well known. However, no reasonable variation of the phase shifts can make α/C much less than one unless pion-pion terms are included. At the same time variation in both α and C may be large. In the resonance region the dispersion analysis is less sensitive to the small P-wave phase shifts and the pion-pion terms, so that $\alpha/C\simeq1$ in the resonance region is in agreement with our measurement. The fact that, as shown in Fig. 8, α/C does not assume a value of less than 0.7, even with large Λ, makes it unclear that the difficulties in fitting are due to the presence of the ρ or ω. Figure 9 shows calculated values of $-\alpha/\sigma_0=[(\sigma_\perp-\sigma_{\parallel})/(\sigma_\perp+\sigma_{\parallel})][1/\sin^2\theta]$ as a function of energy, for $\Lambda_\rho=\Lambda_\omega=1.0$, 0, and -1.0. Since data were taken at different angles at different energies, theoretical curves for 60°, 90°, and 120° are given in each case. Here it is clear that a major disagreement would occur for very large values of Λ, and all that can be said is that our measurements,

[20] F. Bulos (private communication).
[21] George E. Modesitt, thesis, University of Illinois, 1958 (unpublished).

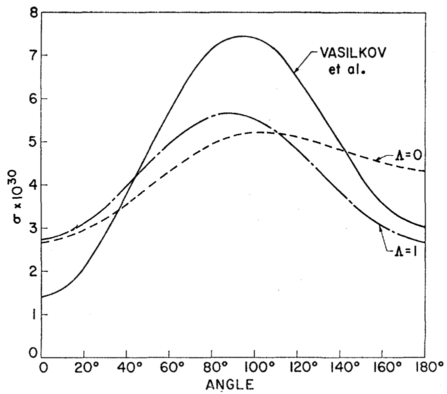

FIG. 10. π^0 photoproduction angular distribution at 235 MeV from CGLN theory. Curves are given for $\Lambda=0$ and $\Lambda=+1$. The Vasilkov *et al.* (Ref. 19) measurements are given for comparison.

if not combined with unpolarized measurements, are consistent with values of $\Lambda=0$ or less than ± 1.0.

In order to complete the analysis, the values of the differential cross section at different angles were computed with and without the pion-pion terms. The argument is that, if pion-pion terms are present, they should fit the measured angular distributions at least as well as if they are not present. The stronger statement that they should give a better fit is difficult to make because too many parameters are present in the theory that include pion-pion terms, i.e., the coupling constants, the small phase shifts, and the question of the proper masses. This calculation was carried out at 235 MeV, and the results are shown in Fig. 10. The angular distribution labeled Vasilkov *et al.* has been reconstructed from their quoted angular coefficients at 240 MeV. From this figure we conclude that even the analysis including the pion-pion terms does not fit the measured unpolarized angular distributions.

A new photoproduction theory has been advanced by Salin and Gourdin[16] since the completion of this analysis. Our asymmetry measurements have been cited by them in support of their theory. While exact calculations have not been made, we feel that quite general arguments show that the new theory does not by itself explain the variation with energy of α/σ_0 observed in this experiment and C/σ_0 observed in the unpolarized experiments.

We consider only the 235-, 285-, and 335-MeV data since above resonance qualitative arguments are made difficult by the second resonance and other higher angular-momentum states. Gourdin and Salin have calculated the effects of the 3,3 resonance by considering the resonance as a $\frac{3}{2}$, $\frac{3}{2}$ particle. In the first resonance region the theory thus contains only this graph plus small Born-approximation terms. By approximating the contributions of this graph, they obtain an equation for the ratio E_{13}/M_{13} involving two constants which are then found by comparing the theory with the measured angular distributions. With no pion-pion terms included, they obtain $E_{13}/M_{13}=0.13$ and obtain a better fit to our measured asymmetries with this ratio than with $E_{13}=0$. Although this theory makes $|\alpha/C|<1$ it incorporates an electric quadrupole term which is in exact phase with the resonating M_{13} term. We argue then that this electric quadrupole term has the same energy dependence as the magnetic dipole M_{13} term. Since other small Born terms are present (i.e., M_{11}, E_{01}, and D wave) the effects of such a term are most prominent at resonance so that the value of $|\alpha/C|$ should be even smaller at resonance than below. This is the opposite of the energy dependence shown in Fig. 7.

On the other hand, a pion-pion term with a positive Λ gives the energy dependence of $|\alpha/C|$ observed since it has no phase shift and hence, there can be no contribution at resonance from its interference term with the main M_{13} term.

At this point it seems proper to reiterate our major conclusion. Using our measured value of asymmetry and published values of the angular coefficients, we are in disagreement with a low-energy phenomenological analysis including S and P waves, and assuming no electric quadrupole. Further conclusions are based on the use of dispersion theory, and are thus subject to the theoretical uncertainties of such an analysis, including for example, the contributions to the dispersion intergrals of the high-energy and the unobservable energy regions.

We conclude that the dispersion theory analysis indicates that the combined experimental results cannot be explained well even by including terms such as those due to the pion-pion resonances. However, such terms help the agreement and if one uses them, the dominant coupling constant or constants must be positive to predict $|\alpha/C|$ less than one below resonance. The disagreement with a negative Λ is very striking since a negative value would make $|\alpha/C|\gg 1$ below resonance, and our measured variation is of an opposite nature.

B. Polarization Measurements

The investigations undertaken in this experiment concerned two separate types of measurements. The attitude adopted in those described here was that a phenomenological analysis of the meson asymmetry (or possibly the dispersion theory analysis) should be accurate near 335-MeV photon energy, and so the reaction could be used to investigate the characteristics of the bremsstrahlung beam. As our analysis will indicate, it is difficult to make precise measurements of the polarization because of the theoretical and experimental uncertainties. However, the polarization calculation is based on the well-understood principles of quantum electrodynamics so that major inaccuracies should not be expected if our multiple scattering and aperture folds are accurate.

Taylor[2] has used the π^+ photoproduction reaction as a detection scheme to determine experimentally the manner in which the polarization of the bremsstrahlung beam varies with bremsstrahlung angle and these results are in agreement with the polarization calculations. Intrinsically the π^0 reaction is more suitable as a polarization analyzer because the "retardation" term of charged pion production is absent, so the theoretical analysis is probably more exact. In addition, this experiment utilized a magnetic spectrometer and counting electronics which permitted faster counting rates and lower backgrounds so that the results are more accurate statistically, the faster data rates also permitting measurements at larger bremsstrahlung angles. For these reasons we have repeated the Taylor measurements and extended them to larger bremsstrahlung angles.

Angular-distribution measurements, measurements of the differential cross section for π^0 photoproduction as a function of center-of-mass angle using unpolarized photon beams, have been made by many experimenters in the resonance region and are accurate relative to measurements made at other photon energies because of the large cross section at this energy. These measurements indicate that the differential cross section may be described by a third-order expansion in $\cos\theta$, where θ is the pion center-of-mass angle, i.e.,

$$\sigma_0 = A + B\cos\theta + C\cos^2\theta. \quad (10)$$

The angular coefficients A, B, and C are determined by a least-square fit to the measured angular distributions. Theoretical considerations described above indicate the cross section should perhaps be described by a higher order polynomial in $\cos\theta$, at least by including a term of order $\cos^3\theta$ and possibly more, but because of the dominating resonance such terms were calculated to be small in the resonance region. In any case the subsequent analysis of polarization versus bremsstrahlung angle will depend on such terms only to the extent they influence the coefficients C and A determined from the least-square fit to the angular distributions. As shown below, this is true because our measurements of polarization versus bremsstrahlung angle were taken at $\theta = 90°$ where such terms vanish. Moreover, as was mentioned above, the presence of $\cos^3\theta$ terms will change only the B coefficient and not affect the value of C/A.

Equation (1) may be rewritten

$$P = [(R-1)/(R+1)][(\sigma_\perp + \sigma_{\parallel})/(\sigma_\perp - \sigma_{\parallel})]. \quad (11)$$

In this portion of the experiment where the value of P is to be studied, the asymmetry is regarded as known, and hence, our measurements of R can be used to predict the polarization. P is then compared with its calculated value. To be successful, this procedure obviously relies on a known value of the asymmetry which, in turn, implies a reasonable understanding of the meson physics involved. The analysis of Sec. VI indicates two methods that may be used to predict the values of the asymmetry. The first method is phenomenological based on as S- and P-wave analysis; the second method is more theoretical and ultilizes the relativistic dispersion relations. The two methods are not in complete agreement at the resonant energy, and we consider that this disagreement limits our accuracy about 10%. Equation (6) of Sec. VI at $\theta = 90°$ may be written

$$(\sigma_\perp - \sigma_{\parallel})/(\sigma_\perp + \sigma_{\parallel}) = -\alpha/A, \quad (12)$$

where σ_0 has reduced to A and α is an angular coefficient referring to a term in the polarized cross section [Eq. (5)]. The arguments in Sec. VI indicate that at resonance $\alpha = C$; exactly using the phenomenological S- and P-wave analysis neglecting electric quadrupole production, and approximately using the dispersion theory analysis. The dispersion theory calculation predicts α/C between 0.90 and 1.05 at 335 MeV including terms due to pion-pion interactions with Λ between 0 and +2. These results indicate that the approximation $\alpha/C = 1_{-0.10}^{+0.05}$ might be reasonable in the resonance region. For negative Λ very large changes of α/C are possible even at resonance, but negative values are completely inconsistent with our measurements at other energies.

The polarization for these conditions may therefore be written as

$$P = -\frac{R-1}{R+1}\frac{A}{C} \times (1.0_{-0.10}^{+0.05}), \quad (13)$$

so that by using our measured values of R and published values of A and C the polarization can be predicted. Errors in A and C were combined as if they were statistically independent. Such a procedure slightly overestimates the errors, and results in the value $-A/C = 1.60 \pm 0.10$ at 335-MeV photon energy.

1. Measurement of the Polarization

The measured asymmetry at the 335-MeV point is 0.451 ± 0.026. The average polarization at this point was calculated to be 0.160. The polarizations calculated from the different glass slide measurements of the electron distribution at this point varied about 9% and, since the data are averaged over this polarization, a realistic estimate of the error in the polarization would be $\pm 5\%$ which gives $P = 0.160 \pm 0.008$ as the calculated value of polarization for this point. Using the asymmetry predicted by the experimentally measured photoproduction coefficient and the simple phenomenological assumptions of only S and P waves gives $P = 0.144 \pm 0.012$.

That predicted by CGLN dispersion theory gives $P = 0.16 \pm 0.009$ quite independent of pion-pion terms for reasonable values of Λ.

Using the dispersion theory of McKinley we obtain

$$P = 0.144 \pm 0.008 \quad \text{for} \quad \Lambda = 0.$$

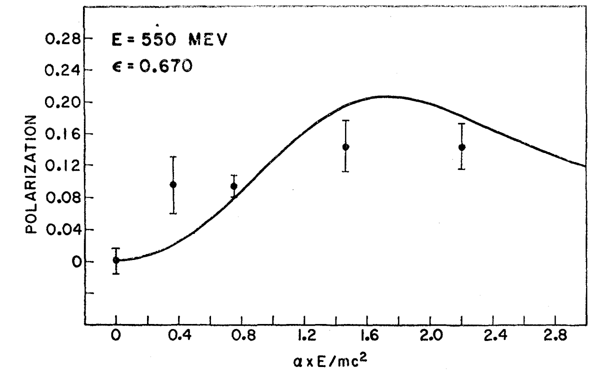

FIG. 11. Polarization as a function of angle at a peak beam energy of 550 MeV. The curve is calculated from formulas (2) and (3). The experimental points are plotted assuming that $\alpha/C=1$, and that there is no pion-pair contamination.

Variations of Λ_p between ± 1 vary P by -0.003 and $+0.004$, respectively, and inclusion of the ω term in CGLN theory causes even less variation. We conclude that the resultant numbers are in statistical agreement with the calculated polarization.

2. *Relative Polarization versus Bremsstrahlung Angle Measurements*

An examination of the relevant kinematics of the processes $\gamma+p\rightarrow\pi^0+p$, and $\gamma+p\rightarrow\pi^0+\pi^0+p$ or $\gamma+p\rightarrow\pi^++\pi^-+p$ revealed it would be difficult to perform these measurements in such conditions that the recoil protons from pion-pair production could be kinematically excluded and still maintain sufficient polarization in the beam to give meaningful results when the spectrometer was set to observe the recoil protons from π^0 photoproduction at the required conditions. (The polarization at a given photon energy increases as the electron beam energy is increased above this value and photons of all energies below the electron energy are present in the bremsstrahlung beam.) As a consequence, it was decided to take data with conditions allowing some pion-pair contamination. Data were taken with the photon energy 335 MeV and two peak beam energies 550 and 850 MeV. In the first case a small amount, $<3\%$, of pion-pair contamination was possible, and in the second, there was obviously a lot. Asymmetry effects of this contamination, while unknown, should be smaller than the asymmetry effects due to single pion production because the photons producing the pairs are of higher energy and consequently, lower polarization than those in the 335-MeV region. One would therefore expect such a contamination to reduce the measured value of R, and decrease the value of P found from Eq. (13). If we assume that near threshold one pion comes off in a P state, the other in an S state, the extra pion adds a pseudoscalar to the reaction. Consequently, one would expect protons from pion pair contamination to have an asymmetry opposite to the single pion production asymmetry. This effect would also decrease our measured value of R. Since we compare P from Eq. (13) with a value of P computed from the polarization calculation described previously, the computed curves for the 850-MeV beam energy should be higher than the experimental points and, as a result, these measurements are to be interpreted only as relative answers. However, whatever the correct value of the asymmetry is, it remains constant in these measurements. Furthermore, inaccuracies in the correct magnet setting, counter voltages, discriminator settings, etc., to properly identify protons from π^0 photoproduction at the correct energy and angle are unimportant since they, too, remain nearly constant in the experiment. The measurement is thus a measure of the relative value of polarization as a function of bremsstrahlung angle. Since an absolute determination of the polarization as a function of bremsstrahlung angle was unreliable with these measurements even with 550-MeV peak beam energy where a small pion-pair contamination was possible, the data were analyzed using two methods. The first method used the angular coefficients in the form $-C/A$ as the value of asymmetry to be used in Eqs. (11) and (13). The results of this analysis are shown in Figs. 11 and 12.

For the data taken at the two electron energies, 850 and 550 MeV, the collimator used corresponded to aperture diameters of 1.56 mc^2/E and 1.01 mc^2/E, respectively. The values of the parameter ϵ at 335-MeV photon energy are 0.609 and 0.394. The results of the polarization calculation for these conditions are plotted as the curves on these two figures. The curves shown are calculated using representative values of the electron distribution at the two energies. The representative value at 550 MeV was determined by the glass-slide measurements of Smith,[3] and 850 MeV by the average of our glass-slide measurements. The beam spot was carefully aligned and focused so as to maintain it as nearly constant as possible on the different nights the runs were taken. The glass-slide measurements at 850 MeV indicated that the measured electron

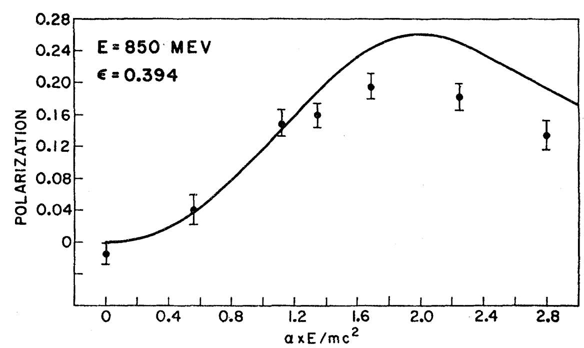

FIG. 12. Polarization as a function of angle at a peak beam energy of 850 MeV. The curve is calculated from formulas (2) and (3). The experimental points are plotted assuming that $\alpha/C=1$, and that there is no pion-pair contamination.

distribution varied only slightly from the chosen "typical" value. An attempt was made to take data with nearly equal statistical accuracy at all values of bremsstrahlung angle on each night's run, and the averaged data then represent the results of an average polarization. It was not feasible to follow this procedure exactly, especially at the larger values of bremsstrahlung angle taken at 850 MeV, so the computed photon polarizations may be slightly incorrect at these points. In addition, the polarization is relatively insensitive to multiple scattering at these large angles.

The second method of analysis assumed that the meson asymmetry is unknown because of the possible pion-pair contamination in the data. While the meson asymmetry is unknown it must be constant for all of the measurements at a given peak bremsstrahlung energy. This means that each point of the data shown in Figs. 11 and 12 can be wrong only by a constant factor, if the previous assumption is correct that the polarization changes due to the different electron distributions are small. Consequently, the data were normalized, choosing that scale factor which minimized chi square for each curve. The resultant χ^2 was then used to determine at what confidence level the data fitted the predicted curve. This "minimum χ^2 fit" is shown in Figs. 13 and 14. In Fig. 14 the dashed curves show effects of the range of polarization possible during this test. The value of χ^2 determined by this analysis is too large since no attempt has been made to include errors in the value of the bremsstrahlung angle. In addition, the data were taken on several nights and the comparison, as mentioned above, is made with the averaged value of polarization obtained for these runs rather than the individual values for each night. The analysis does not indicate disagreement between the computed curves and meson normalized data. As can be seen from Figs. 13 and 14 the data are in appreciably better agreement at 850 MeV than at 550 MeV. The chi-square analysis indicates agreement at a 10% confidence level at 850 MeV and at a 4% confidence level at 550 MeV.

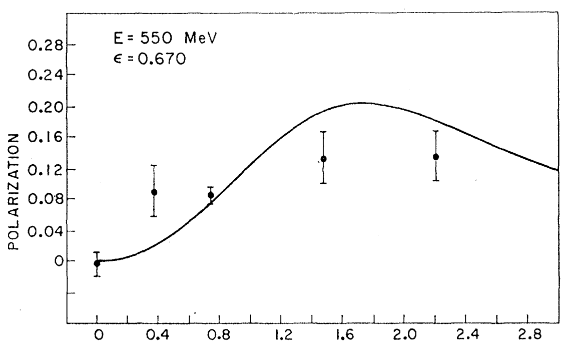

FIG. 13. Polarization as a function of angle for a peak beam energy of 550 MeV. This is the same as Fig. 11 except that the data are adjusted to give a best fit to the calculated curve using a constant normalizing factor to take into account the possibility of pion-pair contamination.

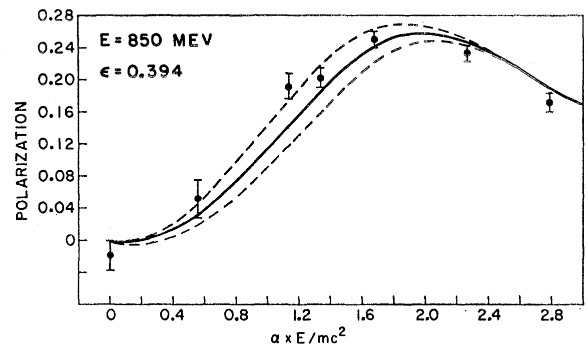

FIG. 14. Polarization as a function of angle for a peak beam energy of 850 MeV. This is the same as Fig. 12 except that the data are adjusted to give a best fit to the calculated curve using a constant normalizing factor to take into account the possibility of pion-pair contamination. The dashed lines show the range of polarization possible during the periods in which these data were taken.

We would have hoped for a better agreement since such inconclusive results only allow us to state that our measurements are compatible with the theory.

A summary of our results is then that the measured value of polarization and its change with angle is consistent with the calculated value. If we use the calculated value to study π^0 production, our measurements when coupled with those of Vasilkov *et al.* give results which cannot be easily explained in terms of a simple *S*- and *P*-wave production or dispersion theoretical calculations. The introduction of pion-pion vertices with positive coupling helps bring to the data into better agreement but is not completely effective.

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